Articles

The Farm-Retail Price Spread in a Competitive Food Industry

Bruce L. Gardner

Consistency with market equilibrium places constraints on the pricing policies of food marketing firms in a competitive industry. This paper examines the implications of simultaneous equilibrium in three related markets: retail food, farm output, and marketing services. From equations representing the demand and supply sides of each market, elasticities are generated which show how the farm-retail price spread changes when retail food demand, farm product supply, or the supply function of marketing services shifts. Implications for the viability of simple markup pricing rules and the determinants of the farmer's share of the food dollar are discussed.

Key words: farm-retail price spread, marketing margin, market equilibrium, competition.

This study examines the consequences of competitive equilibrium in product and factor markets for the relationship between farm and retail food prices. The investigation is based on a one-product, two-input model developed by Allen and Hicks and since applied to many issues at the industry level. Notable agricultural examples are the papers of Brandow and Floyd. The model is used in this paper to generate quantifiable predictions about how various shifts in the demand for and supply of food will affect the retail-farm price ratio and the farmer's share of retail food expenditures. The results have implications for the viability of simple rules of markup pricing by marketing firms. In general, the markup must change whenever demand or supply shifts in order to be compatible with market equilibrium. Moreover, the markup will be forced to change in different ways depending on whether price movements originate from the retail demand or farm supply side. Related implications concern the consequences of retail price ceilings and farm price floors, the elasticity of price transmission, and the determinants of changes in the farmer's share of the food dollar.

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The Model

Consider a competitive food marketing industry using two factors of production, purchased agricultural commodities (a) and other marketing inputs (b), to produce food sold at retail (x). The marketing industry's production function is

$$x = f(a,b). \tag{1}$$

It is assumed to yield constant returns to scale. The retail food demand function is

$$x = D(P_x,N), \tag{2}$$

where P_x is the retail price of food and N is an arbitrary exogenous demand shifter which for purposes of specificity will be called population.

The model is completed by equations representing the markets for b and a. On the demand side, firms are assumed to want to buy the profit-maximizing quantities of b and a, which implies that value of marginal product equals price for both

$$P_b = P_x \cdot f_b \tag{3}$$

and

$$P_a = P_x \cdot f_a, \tag{4}$$

where f_b and f_a are the partial derivatives of x with respect to b and a.

The input supply equations are

(5) $$P_b = g(b,T),$$

the supply function of b to the food marketing industry, and

(6) $$P_a = h(a,W),$$

the supply function of agricultural output. The exogenous shifters of marketing input and farm product supply are represented by T and W. For purposes of specificity, W may be thought of as a weather variable for which higher values increase P_a (e.g., an index of drought), and T as a specific tax on marketing inputs which makes them all more expensive.

This system contains six equations in six endogenous variables (x,b,a,P_x,P_b,P_a). Under normal conditions (where the demand function has negative and the supply functions have nonnegative slopes), there will be a unique equilibrium for given values of the exogenous variables. At this equilibrium, the values of the six endogenous variables, and hence the farm-retail spread, are determined. This price spread may be measured by the difference between the retail and farm price, $P_x - P_a$, by the ratio of the prices, P_x/P_a, by the farmer's share of the food dollar, aP_a/xP_x, or by the percentage marketing margin, $(P_x - P_a)/P_a$. This paper focuses on the retail-farm price ratio, the closely related percentage margin, $P_x/P_a - 1$, and the farmer's share of retail food expenditures.

Effect of a Food Demand Shift on the Retail-Farm Price Ratio

The effects of a shift in retail demand on market equilibrium are analyzed by differentiating equations (1) to (6) with respect to N, while W and T are held constant. The six equations can be immediately reduced to three (one equation for the final product market and one for each input) by equating (1) and (2) to eliminate x, (3) and (5) to eliminate P_b, and (4) and (6) to eliminate P_a.

Beginning with the market for a, equations (4) and (6), the new equation is

(7) $$h_a \frac{da}{dN} = P_x \frac{df_a}{dN} + f_a \frac{dP_x}{dN}.$$

The df_a term of equation (7) must be expanded further. It is not simply the second partial derivative of x with respect to a (which will be written f_{aa}). It also brings in the amount of b that a has to work with as

(8) $$\frac{df_a}{dN} = f_{aa} \frac{da}{dN} + f_{ab} \frac{db}{dN}.$$

Substituting equation (8) into (7) yields

(9) $$h_a \frac{da}{dN} = P_x f_{aa} \frac{da}{dN} + P_x f_{ab} \frac{db}{dN} + f_a \frac{dP_x}{dN}.$$

Next, analyze the b market by combining equations (3) and (5) and differentiating:

(10) $$g_b \frac{db}{dN} = P_x f_{bb} \frac{db}{dN} + P_x f_{ba} \frac{da}{dN} + f_b \frac{dP_x}{dN}.$$

Equation (10) holds the b market in equilibrium while the relationship between dP_x and dP_a is examined. Similarly, the third equation specifies equilibrium in the x market by differentiating equations (1) and (2) combined:

(11) $$f_a \frac{da}{dN} + f_b \frac{db}{dN} = D_{P_x} \frac{dP_x}{dN} + D_N.$$

Equations (9)-(11) can be solved for da/dN, db/dN, and dP_x/dN. The solution is made more intelligible by converting all derivatives to elasticities. Details of the necessary manipulations are presented in an appendix. The result is the three equation system:

(12) $$0 = -\left(\frac{S_b}{\sigma} + \frac{1}{e_a}\right) E_{aN} + \frac{S_b}{\sigma} E_{bN} + E_{P_xN},$$

(13) $$0 = \frac{S_a}{\sigma} E_{aN} - \left(\frac{S_a}{\sigma} + \frac{1}{e_b}\right) E_{bN} + E_{P_xN},$$

and

(14) $$\eta_N = S_a E_{aN} + S_b E_{bN} - \eta E_{P_xN}.$$

Equation (12) pertains to the market for a, (13) to b, and (14) to x; S_a and S_b are the relative shares of a and b, e.g., $S_a = aP_a/xP_x$; σ is the elasticity of substitution between a and b; η is the price elasticity of demand for x; e_a and e_b are the own price elasticities of supply of a and b; η_N is the elasticity of demand for x with respect to N; and E_{aN}, E_{bN}, and E_{P_xN} are total elasticities which tell how the first subscripted variable responds to a change in the second.[1]

[1] The capital E's are elasticities which take into account equilibrating adjustments in all three markets simultaneously; e_a, e_b, and η are partial elasticities which refer to movements along the input supply and product demand functions. All the elasticities

The question to be investigated is how P_x/P_a changes when the demand for food shifts. The answer can be expressed as the elasticity of P_x/P_a with respect to N. This elasticity is equal to the difference between E_{P_xN} and E_{P_aN}, both of which can be obtained from the system of equations (12) to (14). The result (derived in the appendix) is

$$(15) \qquad E_{P_x/P_aN} = \frac{\eta_N S_b(e_a - e_b)}{D},$$

where D is a function of σ, η, e_a, e_b, and S_a.[2] The denominator has no intuitively clear meaning but is positive in all normal cases ($\eta < 0$ and e_a and $e_b \geqslant 0$). Therefore, the numerator normally determines the sign of equation (15).

Because of the way the original model was constructed, equation (15) will be more readily applicable to some situations than others. In reality, of course, there are many marketing activities and many marketing inputs. The present model assumes that these can all be lumped together into a single production function with a single marketing input, b. Following the usual requirements for aggregation, this assumption should cause no analytical difficulties so long as the relative prices of the components of b are constant. Thus, equation (15) will be helpful in understanding how shifts in food demand affect agricultural product prices relative to all marketing inputs as a group, but will not be helpful in situations where substantial relative price changes within the set of marketing inputs are induced.

There may also be an aggregation problem with the quantity of retail food, x, depending on the context in which the model is applied. If x is taken to be an aggregate of all food, it must be assumed that the relative prices of the various food products are held constant. Thus, the exogenous shift in demand should be thought of as one applying to all forms of food.

On the other hand, if the context in which the model is applied is a relatively narrowly defined product, say, wheat, the aggregation problems for both x and b may be less serious.[3] For a case like wheat as the farm product and bread as the retail product, what is the probable sign of equation (15)? Since wheat is a specific factor to the x industry, while the components of b (labor, transportation, packaging, etc.) generally are not, and since a is land intensive, it seems likely that $e_a < e_b$. In this case, when the demand for food shifts to the right, P_x/P_a falls. Therefore, the retail-farm price ratio is expected to decline when population (or any other exogenous food demand shifter) increases.

An interesting special case arises when $e_a = e_b$. In this case P_x/P_a is unchanged when the demand for food shifts. Thus, a fixed percentage markup rule used by marketing firms is viable in the sense that competitive forces will not require the markup to change when retail food demand shifts. In general, however, $e_a \neq e_b$ and a fixed percentage markup will not be viable in this sense.

Equation (15) also helps in understanding the role of σ, the elasticity of substitution between a and b in the marketing industry. Suppose N increases, and $e_a < e_b$. Then the price of raw farm product relative to marketing inputs increases, creating an incentive to substitute the latter for the former. In the wheat example, additional labor may be used to reduce grain wastage in processing operations, and the use of pest and spoilage control may increase.[4] However, in many marketing contexts the opportunities for substitution appear limited. This would be reflected in equation (15) by a small value of σ. Since σ appears only in the denominator and with a positive sign, the smaller σ is the more volatile the retail-farm price ratio.

The economic reason for this result can be illustrated with reference to an increase in retail demand for food. The demand shift increases the derived demand for both farm products and the nonagricultural inputs used in the food marketing industry. But so long as the two elasticities of supply are different ($e_a \neq e_b$), their relative prices must change. How much P_a/P_b will change depends on the degree to which a and b can be substituted in the

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are partial in the sense that the exogenous variables T and W are held constant.

[2] $D = -\eta(S_b e_a + S_a e_b + \sigma) + e_a e_b + \sigma(S_a e_a + S_b e_b)$.

[3] The model applied separately to each of a set of narrowly defined retail products would still have implications for the average or aggregate farm-retail price spread. For example, a shift in demand towards relatively b-intensive products (such as TV dinners) would reduce the aggregate farmer's share even if the share for any particular product remained unchanged.

[4] Analytically troublesome issues are raised by the possibility of changing the nature of the product when P_a/P_b rises, for instance, economizing on wheat use by milling poorer quality wheat or even introducing a bit of sawdust into the cracked wheat bread. One may question in such a case whether our observations are of movements along a well-defined demand curve and production function. This is not, of course, a difficulty peculiar to the present model. It pertains to almost any situation in which substitution in production is possible. Moreover, if we were purist enough to say that the nature of a food product changed whenever its farm level price changed, it could also put a quick end to empirical studies of retail food demand.

marketing process. The greater σ is, the less P_a/P_b will change when P_x is changing. In the extreme case when $\sigma \to \infty$, equation (15) approaches zero and P_x/P_a is constant.

In the more realistic limiting case in which $\sigma \to 0$, the Marshallian derived demand model applies (Friedman, chap. 7). In this case the propositions concerning e_a, e_b, and η in this and the following sections can be derived graphically using the methods of Tomek and Robinson (chap. 6).

Effect of a Farm Product Supply Shift on the Retail-Farm Price Ratio

A shift in equation (6) is analyzed by taking derivatives with respect to W, while dN and dT are held equal to zero. When the results are converted to elasticities, a system of three equations identical to equations (12) to (14) results, except that all E's have W as their second subscript; η_N becomes zero in equation (14), and e_W (the elasticity of P_a with respect to W) replaces zero in equation (12).

Solving this new system for the elasticity of P_x/P_a with respect to a change in W yields[5]

$$(16) \qquad E_{P_x/P_a,W} = \frac{e_W S_b e_a(\eta - e_b)}{D}.$$

Equation (16) differs from equation (15) in that for all normal cases, $E_{P_x/P_a,W}$ is negative. Thus, the percentage difference between P_x and P_a will fall when P_a rises as a result of a leftward shift in the supply function of agricultural output. Conversely, an exogenous event that reduces P_a by increasing a, such as a technical improvement in crop production, will widen the percentage difference between P_x and P_a. The economic reason for this result can be explained as follows. When farm product supply shifts to the right, both P_x and P_a will tend to fall. But the increase in x will require additional marketing inputs. So long as $0 \leq e_b < \infty$, P_b must therefore rise, increasing the cost of marketing relative to farm inputs and hence the ratio P_x/P_a.

As was the case in equation (15), σ plays a moderating role in that the larger σ is, the less a given shift in W will change P_x/P_a.

The responsiveness of P_x/P_a to W varies substantially with the context being considered. For example, in a very short-run context for a narrowly defined product, capacity constraints in marketing activities may make e_b quite small, so that P_x/P_a is especially volatile. Another extreme case would be (external) economies of scale in marketing activities, which would make $e_b < 0$ and could even reverse the sign of equation (16). In this case, an increase in farm product supply could conceivably reduce P_x/P_a by reducing the price of marketing services as output increases. A final interesting special case is that in which marketing inputs are perfectly elastic in supply (a long-run, nonspecific factor case). In this case, P_b remains constant, but an increase in farm supply will still increase P_x/P_a. This occurs because even though P_b is absolutely unchanged, it is increased relative to P_a. Hence the relative contribution to retail food costs accounted for by marketing inputs will increase.

Effect of a Marketing Input Supply Shift on the Retail-Farm Price Ratio

A shift in equation (5) is analyzed by taking derivatives with respect to T, while dN and dW are equal to zero. In this case, the system of equations corresponding to equations (12) to (14) is changed as follows: η_N becomes zero in equation (14), e_T (the elasticity of P_b with respect to T) replaces zero in equation (13), and all E's have T as their second subscript. Solving this system for the elasticity of P_x/P_a with respect to T yields

$$(17) \qquad E_{P_x/P_a,T} = \frac{e_T S_b e_b(e_a - \eta)}{D}.$$

Equation (17) has the same form as equation (16) except that e_a and e_b are interchanged in the numerator and the sign is reversed. Equation (17) will be positive in all normal cases, so that the percentage margin between P_x and P_a will increase when P_b rises as a result of a specific tax on marketing inputs. Thus, while an exogenous change that decreases agricultural supply will decrease the retail-farm price ratio, the same kind of change in the supply of marketing inputs will increase the ratio.

Equation (17) seems more limited in its applicability than equations (16) or (15) due to the aggregation problem. It is difficult to think of exogenous shifters of marketing input supply that will affect all the components of b proportionally. Technical progress, for example, will typically be associated with a particular marketing input or activity. This will

[5] Derivation outlined in appendix.

Table 1. Elasticities of P_x/P_a with Respect to Shifts in Retail Food Demand, Farm Product Supply, and Marketing Input Supply

Parameter Values					$E_{P_x/P_a,N}$ eq. (15)[a]	$E_{P_x/P_a,W}$ eq. (16)[a]	$E_{P_x/P_a,T}$ eq. (17)[a]
σ	e_a	e_b	η	S_b			
0.5	1.0	2.0	−0.5	0.5	−0.13	−0.33	0.40
0	1.0	2.0	−0.5	0.5	−0.18	−0.46	0.54
0	1.5	2.0	−0.5	0.5	−0.06	−0.48	0.52
0	2.0	2.0	−0.5	0.5	0	−0.50	0.50
0	2.0	1.0	−0.5	0.5	0.18	−0.54	0.46
0	1.0	2.0	−1.0	0.5	−0.14	−0.43	0.57

[a] The values of η_N in eq. (15), e_W in eq. (16), and e_T in eq. (17) are set equal to 1. Thus, the elasticity of P_x/P_a with respect to N measures the percentage response in P_x/P_a to a change in N sufficient to shift the demand for x by 1% at given prices.

change the relative prices of the components of b, hence violating a necessary condition for aggregation.

To examine further the anatomy of equations (15), (16), and (17), they can be evaluated at hypothetical parameter values. Let $S_a = 0.5, \eta = -0.5, e_a = 1.0, \sigma = 0.5$, and $e_b = 2.0$. The resulting values of P_x/P_a from equations (15) to (17) are shown in the first line of table 1. The −0.13 elasticity means that a change in population sufficient to generate a 10% rightward shift in retail demand reduces P_x/P_a by approximately 1.3%.[6] Thus, the price ratio (and percentage marketing margin) fall, though quantitatively the response is small.[7]

Keeping the other parameter values the same, let σ be zero. In this case the change in the marketing margin is larger (line 2 of table 1). The economic reason for this result was discussed above with reference to a shift in retail demand.

A crop like sweet potatoes, which uses a relatively small fraction of the land suitable for it, may have e_a larger than 1.0, especially in a long-run context. Lines 3 and 4 of table 1 examine what happens when e_a increases, holding the other parameters constant. From line 5, when $e_a > e_b$, the percentage margin increases when P_a and P_x rise. In this case, when retail demand increases, it is the nonagricultural inputs in marketing that become relatively scarce. However, when the increase in P_a and P_x is induced from a shift in the agricultural supply function, equation (16), P_x/P_a falls when prices increase no matter what e_a is. In this case there is given a change in P_a, say, induced by drought. Of course, e_a enters indirectly in that the larger e_a is, the worse the drought will have to be to obtain a given increase in P_a. (For example, the effect on the price of chickens when several million contaminated birds were killed in Mississippi depended on the degree to which other chicken producers could increase supply in response.) Line 6 shows the consequences of a more elastic demand curve at the retail level, the other parameters remaining the same as in line 2.

Price Supports and Price Ceilings

Price Control on x

If a price ceiling lower than the market-clearing price is imposed on a food product at the retail level, but not at the farm level, what effect will this have on the retail-farm price ratio? This question can be answered by introducing P_x as an exogenous variable in place of the demand equation (2). The resulting system can be solved to obtain

$$(18) \qquad E_{P_a\bar{P}_x} = \frac{\sigma + e_b}{\sigma + S_a e_b + S_b e_a},$$

where $\bar{P}_x$ equals the legal maximum price.[8] If $e_a = e_b$, then $E_{P_a\bar{P}_x} = 1$, and a legislated reduction in P_x will reduce P_a by the same

[6] An approximation because equations (15), (16), and (17) pertain to small changes. The approximation for large changes would be better the closer equations (2), (5), and (6) are to constant own-price elasticity, i.e., log-linear form, and the closer equation (1) is to a CES form.

[7] If the marketing margin is expressed as a percentage markup over the farm level price, then P_x/P_a and the margin are directly related as $P_x/P_a = 1 +$ marketing margin.

[8] No derivation of equation (18) is given because this same result is presented in Floyd (p. 151) to show the effect of a farm level price support on the price of land and labor in agriculture. Even though Floyd considers a minimum price and equation (18) a legal maximum price, the results are the same because in both cases the restriction moves us along the product supply curve. Floyd accomplishes this for the minimum price by having the government buy all that is offered at the support price. In the maximum price case, excess demand will exist at $\bar{P}_x$, requiring nonprice rationing.

percentage. In this case, the percentage marketing margin is unchanged. It seems likely, however, that $e_a < e_b$, which implies that $E_{P_a P_x} > 1$. In this case, P_a falls by a greater percentage than P_x so that the percentage margin widens when price controls are imposed.

The sign of equation (18) is positive, implying that an effective price ceiling on retail food will always reduce farm level prices. The only exception would be if $e_a \to \infty$. In this case, a price ceiling on P_x would leave P_a unchanged. The reason for this result is that the price ceiling on x always reduces the derived demand for a, even though consumers want to buy more x at the lower P_x. Derived demand is reduced because competitive marketing firms cannot afford to pay as much for a with the price ceiling as they could without one. The exception when $e_a \to \infty$ arises because when a is perfectly elastic in supply, its price is unaffected by a shift in the derived demand for a.

Price Control on a

If the price of a is kept at a legislated level by means of production controls, what effect will this have on the retail-farm price ratio? This question can be answered by leaving out the supply equation (6) and introducing a as an exogenous variable. The resulting system can be solved to obtain

$$(19)\qquad E_{P_x \bar{P}_a} = \frac{S_a(\sigma + e_b)}{e_b + S_a\sigma - S_b\eta},$$

where $\bar{P}_a$ is the legislated price support level. In order for a percentage marketing margin to remain unchanged, $E_{P_x \bar{P}_a}$ must equal 1. As long as $e_b > \eta$, that is, in all normal cases, equation (19) will be less than 1.[9] Therefore, a production control program that raises P_a will raise P_x by a smaller percentage, and the percentage margin will narrow.

The Elasticity of Price Transmission and the Elasticity of Derived Demand

The percentage change in P_x associated with a change in P_a is equal to the reciprocal of equation (18) when the change originates in the x market and to equation (19) when the change originates in the a market (see equations [A.10–11]). Since equations (18) and (19) are different, the value of the elasticity of price transmission is obviously not independent of whether the exogenous changes that generate our observations come from the demand for x or the supply of a. If the supply of a is the source of observed price changes, then equation (19) applies, and $E_{P_x P_a}$ is less than one. But if shifts in food demand are responsible for observed price changes, equation (18) applies and $E_{P_x P_a}$ will be closer to unity, and will exceed it if $e_a > e_b$, i.e., if marketing inputs are more nearly fixed in supply than are farm products.

A function such as George and King's (p. 57),

$$P_a = \alpha + \beta P_x,$$

even if it fits perfectly conditions generated by farm supply shifts, would not yield estimates of α and β applicable to conditions generated by retail demand shifts. Estimation when both farm supply and retail demand are shifting would yield an elasticity of price transmission that is a hybrid of equations (18) and (19).[10]

George and King use the elasticity of price transmission to derive farm-level elasticities from retail price elasticities of demand. In the terminology of this paper, their result (p. 61) is

$$(20)\qquad E_{aP_a} = (\eta)(E_{P_x P_a}).$$

This equation can be misleading. The problem is that George and King, following Hildreth and Jarrett (p. 108), do not distinguish between quantities of product at the farm and retail level. They assume that $x \equiv a$. This assumption is of no great analytical significance in the case of fixed proportions, since a can be transformed into x by means of a constant production coefficient. Although fixed proportions may not be an unreasonable assumption in many marketing contexts, there are several commodities examined by George

[9] The condition for equation (19) being less than 1 is

$$s_a\sigma + S_a e_b < e_b + S_a\sigma - S_b\eta.$$

Subtracting $S_a\sigma + e_b$ from both sides and dividing by $-S_b$ yields $e_b > \eta$.

[10] There is one other $E_{P_x P_a}$ relationship that arises indirectly when the exogenous event that changes both P_x and P_a is a shift in marketing input supply, T in equation (5). This elasticity is

$$E_{P_x P_a} = \frac{\sigma + e_a}{\sigma + \eta}\ (N,\ W \text{ constant}).$$

This case is especially interesting in that it is the only one that can account, within the confines of this model of this paper, for a simultaneous fall in P_a and a rise in P_x. While equations (18) and (19) are positive for all normal parameter values, this elasticity is not. It will be negative whenever $\sigma < |\eta|$.

and King for which the ratio of a to x may vary.

A more general statement of the relationship between the retail elasticity of demand for food, η, and the farm level elasticity of demand, E_{aP_a}, is readily obtainable from the original system of equations (1)–(6). As Floyd (p. 153) shows, the elasticity of demand for a, which is identical to the elasticity of factor demand found by Hicks (p. 244), is

$$E_{aP_a} = \frac{\eta\sigma + e_b(S_a\eta - S_b\sigma)}{e_b + S_a\sigma - S_b\eta}. \tag{21}$$

Whether E_{aP_a} is greater than or less than η depends on the relative size of σ and (the absolute value of) η. The derived demand function for a will be less elastic than the retail demand function if and only if $\sigma < |\eta|$. If $\sigma = |\eta|$, then equation (21) yields $E_{aPa} = \eta$. The retail and farm level elasticities are equal. If $\sigma > |\eta|$, then the derived demand function is more elastic than is the demand function for the final product.[11] In the case of fixed proportions, since $\sigma = 0$, σ is always less than $|\eta|$. Therefore, in this case, farm level demand is always less elastic than retail level demand.

To show how this general approach fits in with the elasticity of price transmission as used by George and King, replace the left-hand side of equation (20) by E_{aP_a} from equation (21). Replace the right-hand side of equation (20) by equation (19) times η. These substitutions yield

$$\frac{\eta\sigma + e_b(S_a\eta - S_b\sigma)}{e_b + S_a\sigma - S_b\eta} \neq \frac{\eta S_a(e_b + \sigma)}{e_b + S_a\sigma - S_b\eta}.$$

In general, the two sides are not equal. But if $\sigma \rightarrow 0$, then

$$\frac{S_a e_b \eta}{e_b - S_b\eta} = \frac{S_a e_b \eta}{e_b - S_b\eta}$$

and the George and King approach is correct.

[11] For instance, if $\eta = -0.2$, $\sigma = 0.5$, $S_a = 0.5$, and $e_b = 1$, the value of equation (21) is

$$E_{aP_a} = \frac{0.5(-0.2) + 1(0.5(-0.2) - 0.5(0.5))}{1 + 0.5(0.5) - 0.5(-0.2)} = -0.33.$$

The farm level demand elasticity is substantially greater.

To obtain the general condition for $E_{aP_a} > \eta$, note that this occurs iff $E_{aP_a}/\eta > 1$. From equation (21), this occurs when

$$\eta\sigma + e_b(S_a\eta - S_b\sigma) > \eta(e_b + S_a\sigma - S_b\eta).$$

Subtracting $S_a\sigma + S_a e_b$ from both sides, dividing by S_b, and factoring out σ on the left-hand and η on the right-hand side yields $\sigma > -\eta$.

The Farmer's Share of the Food Dollar

The data generated by the U.S. Department of Agriculture on farm-retail price spreads do not distinguish between the price ratio P_a/P_x and relative share aP_a/xP_x (which is S_a in the notation of this paper). The two are the same in the USDA publications because the quantities of farm product are adjusted by means of estimated production coefficients to obtain equivalent units for a and x. Thus, P_a is the value of farm product per unit of x. For example, in the case of pork, the farm price for 1969 is multiplied by 1.97 on the grounds that 1.97 pounds of "live hog equivalent" yields 1 pound of pork sold to consumers (Scott and Badger, p. 115). This substitution of units of x for units of a is strictly correct only in the fixed proportions case.[12] In general, the farmer's share of the food dollar is conceptually quite different from the farm price as a percentage of the retail price of food. This share can be analyzed by the same methods used above to analyze P_x/P_a. It turns out (derivation in appendix) that

$$E_{S_aN} = \frac{\eta_N S_b(e_a - e_b)(\sigma - 1)}{D}, \tag{22}$$

where the parameters and D are as defined in equation (15). Since $D > 0$, the numerator determines the sign of equation (22). There are three interesting cases. (*a*) If either $e_b = e_a$ or $\sigma = 1$ (the Cobb-Douglas case), then S_a is constant. A shift in demand for food at the retail level will have no effect on the farmer's share. (*b*) If $e_b > e_a$ and $\sigma < 1$ or if $e_b < e_a$ and $\sigma > 1$, then S_a increases with N. An increase in demand for food will increase the farmer's share. (*c*) If $e_b > e_a$ and $\sigma > 1$ or if $e_b < e_a$ and $\sigma < 1$, then an increase in the demand for food will decrease the farmer's share.

It seems most likely for any particular food commodity or for an aggregate of such commodities that $e_b > e_a$ (the elasticity of the supply curve of agricultural output is less than that of nonagricultural inputs used in the food marketing industry) and $\sigma < 1$. These are case (*b*) conditions, suggesting that the farmer's share should increase in the presence of an exogenous increase in food demand, such as has been created for U.S. farm products by increasing export demand in recent years.

Equation (22) is distinct from, though closely related to, the effect of a change in N

[12] Pork does not seem to constitute a fixed proportions case since the 1.97 figure changes from year to year.

on P_x/P_a as given by equation (15). Equation (22) and the negative of equation (15) are the same if and only if $\sigma = 0$.[13]

Similar methods can be used to analyze the effects of supply as well as demand shifts. The right-hand-side elasticities are different in this case, being derived by differentiating with respect to W instead of N. The resulting equation is

$$(23) \qquad E_{S_aW} = \frac{e_W e_a S_b(\eta - e_b)(\sigma - 1)}{D}.$$

The sign of equation (23) is determined by σ being less than, equal to, or greater than 1. If $\sigma < 1$, then a shift in the supply function of a which increases P_a, for example, a drought, will increase the farmer's share.

The economic sense of this result can be explained as follows. A drought reduces the food supply and hence tends to increase the price of food at both the farm and retail levels. The drought also makes agricultural output scarce relative to marketing inputs. The price of the latter rises by a smaller amount than does P_a. Therefore, the price of retail food rises by a smaller percentage than does the farm level price. If $\sigma \neq 0$ the ratio b/a will increase. The larger σ is, the more the demand for b will shift to the right, and consequently the larger the nonfarm input into food, which implies a smaller relative share of a in retail food costs. The elasticity of supply of b enters because although substitutability of b for a generates a shift in demand for b, the amount of additional b used depends also on its elasticity of supply to the marketing industry.

The preceding discussion is intended to bring out analytical differences between the farmer's share of the food dollar S_a and the price ratio P_a/P_x. The USDA publications on farm-retail price spreads use the share approach by adjusting P_a such that the units it pertains to are units of x. Whether data on S_a or P_a/P_x are more desirable depends, of course, on the use to which they are to be put.[14] The point of this discussion is that one has to be careful in interpreting farmer's share data in price ratio terms when $\sigma > 0$. For example, consider the historical data on price spreads in vegetable shortening. The farmer's share has decreased from 0.43 in 1947–49 to 0.30 in 1967–69 (Scott and Badger, p. 174), a decline of about 30%, while P_a/P_x has increased about 17% over the same period.[15] How is this possible? It is possible because while P_a (the price of soybeans) increased relative to P_x, other inputs have replaced soybeans to such an extent that S_a has actually fallen. Indeed, an estimate of σ can be obtained by dividing the percentage change S_a by the percentage change in P_x/P_a, since they differ only in being multiplied by $(\sigma - 1)$.[16] Thus, $\sigma - 1 \doteqdot \frac{-0.30}{-0.17}$ and $\sigma \doteqdot 2.8$. This is a very crude estimate and implicitly includes alternative farm products to soybeans in b. This may account for the high value of σ. That P_a/P_x increased while S_a decreased itself implies $\sigma > 1$.

Summary and Conclusion

Consistency with market equilibrium in a competitive food industry puts constraints on the pricing policies of food marketing firms. This paper has investigated the consequences of these constraints for the retail-farm price ratio and the farmer's share of the food dollar.

One implication of the results is that no simple markup pricing rule—a fixed percentage margin, a fixed absolute margin, or a combination of the two—can in general accurately depict the relationship between the farm and retail price. This is so because these prices move together in different ways depending on whether the events that cause the movement arise from a shift in retail demand, farm supply, or the supply of marketing inputs.

Some more specific results concerning the retail-farm price ratio are as follows. (*a*) Events that increase the demand for food will reduce the retail-farm price ratio (and percentage marketing margin) if marketing inputs are more elastic in supply than farm products, but increase P_x/P_a if marketing inputs are less elastic in supply than farm products. (*b*) Events that increase (decrease) the supply of farm products will increase (decrease) P_x/P_a.

[13] Because $E_{P_a/P_x N} = -E_{P_x/P_a N}$.

[14] Actually, it is hard to see that either one has much significance for agricultural policy or welfare issues. For most purposes it would seem more pertinent to look at relative farm income than relative prices or shares.

[15] For P_x, the data are the retail price figures of Scott and Badger (p. 174); for P_a, the price of soybeans as reported by the USDA.

[16] This is true whether the observed changes in S_a and P_x/P_a are generated by shifts on the supply side or the demand side, since both equations (15) and (22), and (16) and (23) differ only by the term $\sigma - 1$. The same result holds for elasticities with respect to T.

(*c*) Events that increase (decrease) the supply of marketing inputs will decrease (increase) P_x/P_a. (*d*) An effective price ceiling on retail food will reduce the price of farm products (unless the supply of farm products is perfectly elastic); P_x/P_a will increase (decrease) if the elasticity of supply of farm products is less (greater) than that of marketing inputs. (*e*) Supporting the price of farm products above the unrestricted market equilibrium level will reduce P_x/P_a.

All the preceding propositions can be derived by graphical methods like those of Tomek and Robinson (chap. 6) under the assumption of fixed proportions in food marketing ($\sigma = 0$). The advantage of the mathematical model is that it allows the treatment of the more general case in which $\sigma \geqslant 0$ and it provides quantifiable results.[17]

Other related results are as follows. (*f*) The farm level demand for agricultural products will be more or less elastic than the retail demand for food as $\sigma \lesseqgtr |\eta|$. (*g*) The percentage price spread is analytically distinct from the farmer's share of the food dollar, and the two will behave differently under changing market conditions unless $\sigma = 0$. If $\sigma = 1$, the farmer's share is constant. If $\sigma > 1$, an increase in the marketing margin will be accompanied by an increase in the farmer's share of the food dollar. Otherwise, lower margins go together with an increased farmer's share. (*h*) The elasticity of substitution between farm products and marketing inputs in producing retail food can be estimated by dividing observed changes in the farmer's share of the food dollar by observed changes in the ratio of farm to retail food prices.

Two limitations of the model are that it assumes competition and that it aggregates all marketing activities into one production function and all nonfarm marketing inputs into one quantity.

In relaxing the assumption of competition, although the constraints imposed by competition would disappear, the behavior of the marketing margin would still not be arbitrary. For example, the price behavior of a profit-maximizing retail food seller with monopoly power could be analyzed by replacing marginal product times input price by marginal revenue product in equations (3) and (4). Then elasticities such as equations (15), (16), and (17) could be solved from the new system. Similarly, monopsony in the purchase of a farm product could be introduced by replacing input price by marginal factor cost.

The aggregation problem is serious in some contexts but negligible in others. It is most serious when the changes being considered have large effects on the relative prices of different marketing inputs. In order to examine particular relative price changes within the set of marketing inputs, a three-input model along the lines of Welch might prove a useful alternative approach.

A possible further extension would be to add separate production functions and profit-maximization equations for different marketing activities. This approach would provide more realism for investigating certain problems but would be costly in terms of complexity and intelligibility, and it seems doubtful whether it would yield any basic changes in the results from the simple model of this paper as expressed in propositions (*a*) through (*h*). But this remains to be seen.

Finally, it might prove interesting to investigate the consequences of technical progress in the marketing industry by introducing exogenous shifters of equation (1). This also could follow the approach of Welch.

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[17] In the strict fixed proportions case, marginal products cannot be calculated and the original system of derivatives breaks down. The correct procedure to get quantitative predictions in this case is to take the limit of equations (15) to (23) as $\sigma \to 0$.

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Appendix

Mathematical Derivations

Derivation of $E_{P_x/P_a,N}$. Starting with equations (9) to (11), first convert all derivatives into elasticities. For example, from equation (5),

$$g_b = \frac{dP_b}{db}.$$

Multiplying by b/b and P_b/P_b,

$$g_b = \left(\frac{dP_b}{db} \cdot \frac{b}{P_b}\right) \frac{P_b}{b} = \frac{1}{e_b} \frac{P_b}{b},$$

where e_b is the own-price elasticity of supply to the industry. Second, use the assumption of constant returns to scale to eliminate all second partials (Allen, p. 343), since

$$f_{ab} = \frac{f_a f_b}{\sigma x}$$

and

$$f_{aa} = \frac{b}{a} \cdot \frac{f_a f_b}{\sigma x}.$$

Third, eliminate f_a and f_b wherever they appear by substituting P_a/P_x and P_b/P_x from equations (3) and (4). Making these substitutions and rearranging terms yields equations (12) to (14).

From the system of equations (12)–(14), first find E_{P_xN} by means of Cramer's Rule. Expanding the appropriate determinants,

$$E_{P_xN} = \eta_N \left(\frac{S_bS_a}{\sigma^2} + \frac{S_b}{\sigma e_b} + \frac{S_a}{\sigma e_a} + \frac{1}{e_a e_b} - \frac{S_bS_a}{\sigma^2}\right) \Big/ -\eta \left(\frac{S_b}{\sigma e_b} + \frac{S_a}{\sigma e_a} + \frac{1}{e_a e_b}\right) + \left(\frac{2S_aS_b}{\sigma} + \frac{S_a^2}{\sigma} + \frac{S_b^2}{\sigma}\right) + \frac{S_a}{e_b} + \frac{S_b}{e_a}.$$

The second bracketed term of the denominator equals $1/\sigma$ (since $S_b = 1 - S_a$). Multiplying the numerator and denominator by $\sigma e_a e_b$ yields

$$\text{(A.1)} \quad E_{P_xN} = \frac{\eta_N(S_b e_a + S_a e_b + \sigma)}{-\eta(S_b e_a + S_a e_b + \sigma) + e_a e_b + \sigma(S_a e_a + S_b e_b)}.$$

The denominator of this expression, in the text, and henceforth in this appendix is denoted by D.

Next, from the same system of equations, solve for E_{aN}.

$$\text{(A.2)} \quad E_{aN} = \frac{\eta_N e_a(e_b + \sigma)}{D}.$$

To get from E_{aN} to E_{P_aN}, divide E_{aN} by e_a, since

$$\text{(A.3)} \quad E_{aN}/e_a = \left(\frac{da}{dN} \cdot \frac{N}{a}\right) \Big/ \left(\frac{da}{dP_a} \cdot \frac{P_a}{a}\right), = \frac{dP_a}{dN} \cdot \frac{N}{P_a} = E_{P_aN}.$$

Finally, to get $E_{P_x/P_a,N}$, note that

$$\text{(A.4)} \quad E_{P_x/P_a,N} = \frac{d(P_x/P_a)}{dN} \cdot \frac{N}{P_x/P_a} = \frac{P_a dP_x - P_x dP_a}{P_a^2\, dN} \cdot \frac{NP_a}{P_x} = \frac{NP_a^2 dP_x}{P_a^2 P_x dN} - \frac{NP_x dP_a}{P_x P_a dN} = E_{P_xN} - E_{P_aN}.$$

Substituting (A.1), (A.2), and (A.3) into (A.4) yields

$$\text{(A.5)} \quad E_{P_x/P_a,N} = \frac{\eta_N S_b(e_a - e_b)}{D},$$

which is text equation (15).

Derivation of $E_{P_x/P_a,W}$. After making the changes in equations (12) – (14) described in the text, solve for

$$\text{(A.6)} \quad E_{P_xW} = \frac{e_W e_a S_a(e_b + \sigma)}{D},$$

and

$$\text{(A.7)} \quad E_{a,W} = \frac{e_W e_a(\eta\sigma + e_b(S_a\eta - S_b\sigma))}{D}.$$

To get from E_{aW} to E_{P_aW}, it is again necessary to divide E_{aW} by the elasticity of a with respect to P_a. But the appropriate elasticity is the elasticity of demand for a, not the supply elasticity as was used in equation (A.3). In the preceding section the demand for x was shifting, which generated movement along the supply curve of a. In this section the supply curve of a is shifting, which generates movement along the demand curve for a. Therefore, to get E_{P_aW}, divide E_{aW} by E_{aP_a} where E_{aP_a} is the elasticity of demand for a. Using the formula for E_{aP_a} given as text equation (21) yields

$$(A.8)\qquad E_{P_aW} = E_{aW}/E_{aP_a} = \frac{e_W e_a(e_b + S_a\sigma - S_b\eta)}{D}.$$

Subtracting E_{P_aW} from E_{P_xW} to get $E_{P_x/P_a,W}$ yields

$$(A.9)\qquad E_{P_x/P_a,W} = \frac{e_W e_a S_b(\eta - e_b)}{D},$$

which is text equation (16).

Derivation of $E_{P_xP_a}$ when price changes are caused by a shift in product demand. This elasticity can be obtained by dividing E_{P_xN}/E_{P_aN}. Using equations (A.1), (A.2), and (A.3),

$$(A.10)\qquad E_{P_xP_a} = \frac{\eta_N(S_be_a + S_ae_b + \sigma)}{D} \cdot \frac{D}{\eta_N(e_b + \sigma)} = \frac{S_be_a + S_ae_b + \sigma}{e_b + \sigma}.$$

This is the reciprocal of text equation (18).

Derivation of $E_{P_xP_a}$ when price changes are caused by a shift in the supply curve of a. This elasticity can be obtained by dividing (A.6) by (A.8),

$$(A.11)\qquad E_{P_xP_a} = \frac{e_We_aS_a(e_b + \sigma)}{D} \cdot \frac{D}{e_We_a(e_b + S_a\sigma - S_b\eta)} = \frac{S_a(e_b + \sigma)}{e_b + S_a\sigma - S_b\eta},$$

which is text equation (19).

Derivation of E_{S_aN}. First consider the total differential of S_a

$$dS_a = \frac{xP_x(adP_a + P_ada) - aP_a(xdP_x + P_xdx)}{(xP_x)^2}.$$

Dividing through by a change in population, dN, to get an exogenous influence on the system from the demand side, yields (after converting to elasticities)

$$(A.12)\qquad E_{S_aN} = E_{P_aN} + E_{aN} - E_{P_xN} - E_{xN}.$$

These elasticities were all discussed earlier except for E_{xN}. Using the facts that $f_a = P_a/P_x$, $f_b = P_b/P_x$, and $dx = f_ada + f_bdb$, this elasticity is analyzed as follows:

$$(A.13)\qquad \begin{aligned} E_{xN} &= \frac{dx}{dN} \cdot \frac{N}{x} = f_a \frac{da}{dN} \cdot \frac{N}{x} + f_b \frac{db}{dN} \cdot \frac{N}{x} \\ &= \frac{P_a}{P_x} \cdot \frac{da}{dN} \cdot \frac{N}{x} \cdot \frac{a}{a} + \frac{P_b}{P_x} \cdot \frac{db}{dN} \cdot \frac{N}{x} \cdot \frac{b}{b} \\ &= S_aE_{aN} + S_bE_{bN}. \end{aligned}$$

Substituting equation (A.13) into equation (A.12) yields

$$(A.14)\qquad E_{S_aN} = E_{P_aN} - E_{P_xN} + S_b(E_{aN} - E_{bN}).$$

The new elasticity in (A.14) is E_{bN}. It is the third variable in the system, equations (12) to (14), which has already been solved for E_{aN} and E_{P_xN}. Returning to the equation system for the first part of the appendix.

$$(A.15)\qquad E_{bN} = \frac{\eta_Ne_b(e_a + \sigma)}{D}.$$

Combining equations (A.3), (A.1), (A.2), and (A.15) according to equation (A.14) yields

$$(A.16)\qquad \begin{aligned} E_{S_aN} &= \frac{\eta_N}{D}(e_b + \sigma - S_be_a - \sigma + S_be_ae_b \\ &\quad + S_be_a\sigma - S_be_be_a - S_be_b\sigma) \\ &= \frac{\eta_N}{D}(S_b(e_a - e_b)(\sigma - 1)), \end{aligned}$$

which is text equation (22).

Derivation of E_{S_aW}. Again, all the elasticities are available except E_{bW}:

$$(A.17)\qquad E_{bW} = \frac{e_We_ae_bS_a(\eta + \sigma)}{D}.$$

Combining equations (A.8), (A.6), (A.7), and (A.17) according to equation (A.14) with W replacing N yields

$$\begin{aligned} E_{S_aW} &= \frac{e_We_a}{D}[e_b + S_a\sigma - S_b\eta - S_ae_b - S_a\sigma + S_b\eta\sigma \\ &\quad + S_b(S_ae_b\eta - S_be_b\sigma - S_ae_b\eta - S_ae_b\sigma)] \\ &= \frac{e_We_aS_b}{D}[(\eta - e_b)(\sigma - 1)], \end{aligned}$$

which is text equation (23).